

## **Dynamic Analysis of a Vibration-Based Energy Harvesting System Using a Triboelectric Nanogenerator**

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### **Abstract**

Vibration-based energy harvesting has received increasing attention as an alternative power source for low-energy electronic devices and wireless sensor systems. Among different harvesting mechanisms, triboelectric nanogenerators (TENGs) have recently emerged as promising candidates, particularly due to their effective performance under low-frequency mechanical excitations. Since ambient vibrations are widely available in practical engineering environments, vibration-driven triboelectric energy harvesters have attracted growing research interest.

In this paper, an analytical dynamic study of a vibration-based energy harvesting system incorporating a triboelectric nanogenerator is presented. The mechanical subsystem is modeled as a single-degree-of-freedom mass–spring–damper system subjected to harmonic base excitation. The governing equations of motion are derived using relative displacement coordinates and analytically solved to obtain the steady-state vibration response. The frequency response function of the system is explicitly formulated, and the influence of key parameters such as excitation frequency and damping ratio is investigated. Although the electrical behavior of the triboelectric nanogenerator is not explicitly modeled, a conceptual electromechanical coupling is considered to relate the vibration amplitude and relative velocity to the energy harvesting potential of the system. The results provide clear physical insight into the dynamic behavior of vibration-driven triboelectric energy harvesters and establish a theoretical basis for future numerical and experimental investigations.

**Keywords:** Vibration energy harvesting; Triboelectric nanogenerator; Dynamic modeling; Frequency response; Base excitation.

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## 1. Introduction

Energy harvesting from ambient sources has been widely investigated as a sustainable solution for powering low-energy electronic devices, particularly in wireless sensor networks and structural health monitoring systems [1–3]. Among available ambient energy sources, mechanical vibrations are especially attractive due to their prevalence in industrial machinery, transportation systems, and civil structures [4].

Conventional vibration-based energy harvesting technologies mainly rely on piezoelectric, electromagnetic, and electrostatic mechanisms [5–7]. Piezoelectric harvesters have been extensively studied because of their relatively high energy density; however, their efficiency often decreases under low-frequency and low-amplitude excitation conditions commonly encountered in real environments [8]. In recent years, triboelectric nanogenerators have emerged as a new class of mechanical energy harvesting devices capable of efficiently converting mechanical motion into electrical energy, particularly under low-frequency excitations [9–11].

Triboelectric nanogenerators operate based on the combined effects of triboelectrification and electrostatic induction [12]. Their advantages include simple structure, lightweight design, flexible material selection, and high output voltage [13]. While many previous studies have focused on material optimization and electrical performance of TENGs [14–16], fewer works have addressed the vibration dynamics of triboelectric energy harvesting systems from a mechanical engineering perspective.

For vibration-driven applications, understanding the dynamic response of the mechanical system is essential, since vibration amplitude and relative velocity directly govern the contact–separation behavior of the triboelectric nanogenerator. Motivated by this need, the present study focuses on the analytical dynamic modeling of a vibration-based triboelectric energy harvesting system subjected to base excitation.

This paper presents a vibration-oriented analytical framework for triboelectric energy harvesting systems, focusing on dynamic modeling and frequency response analysis under base excitation.

## 2. Operating Principle of Triboelectric Nanogenerators

Triboelectric nanogenerators generate electrical energy through periodic contact and separation between two materials with different triboelectric polarities [9]. When the materials come into contact, surface charges are generated due to triboelectrification. During separation, electrostatic induction drives charge transfer through an external circuit.

Several operating modes of triboelectric nanogenerators have been proposed, including contact–separation, sliding, single-electrode, and freestanding modes [12]. Among these, the contact–separation mode is particularly suitable for vibration-based applications due to its compatibility with oscillatory motion and structural simplicity [17].

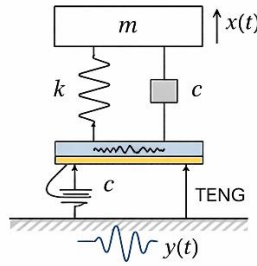
From an electrical standpoint, a triboelectric nanogenerator can be represented by an equivalent circuit consisting of a variable capacitor and a charge source [18]. In vibration-driven systems, the relative motion between mechanical components governs the contact–separation process and therefore directly influences the electrical output. In the present study, a conceptual electromechanical coupling approach is adopted in order to focus on the vibration dynamics without introducing complex electrostatic formulations.

### 3. Dynamic Modeling of the System

In this section, an attempt has been made to present a broad and new view of energy harvesting analysis in new models by presenting the dynamic model and governing equations.

#### 3.1 Mechanical Model

Figure 1 illustrates a schematic representation of the vibration-based triboelectric energy harvesting system. The system is modeled as a single-degree-of-freedom mass–spring–damper system subjected to base excitation.



**Figure 1.** Schematic representation of a vibration-driven triboelectric energy harvesting system modeled as a single-degree-of-freedom mass–spring–damper system under harmonic base excitation.

Let  $m$  denote the equivalent mass,  $k$  the stiffness, and  $c$  the viscous damping coefficient. The base displacement is denoted by  $y(t)$ , while the relative displacement of the mass with respect to the base is denoted by  $x(t)$ . The equation of motion of the system is written as:

$$m \ddot{x}(t) + c \dot{x}(t) + k x(t) = -m \ddot{y}(t) \quad (1)$$

#### 3.2 Harmonic Base Excitation

The base excitation is assumed to be harmonic and is expressed as:

$$y(t) = Y \sin(\omega t) \quad (2)$$

where  $Y$  is the excitation amplitude and  $\omega$  is the excitation frequency. Substituting this expression into the equation of motion yields:

$$m \ddot{x}(t) + c \dot{x}(t) + k x(t) = m \omega^2 Y \sin(\omega t) \quad (3)$$

## 4. Analytical Solution of the Governing Equation

The vibration-based energy harvesting system is governed by the equation of motion of a single-degree-of-freedom system subjected to harmonic base excitation. The absolute displacement of the mass is denoted by  $z(t)$ , while the base displacement is given by  $y(t)$ . The relative displacement between the mass and the base is defined as:

$$x(t) = z(t) - y(t) \quad (4)$$

This relative motion directly drives the contact–separation process of the triboelectric nanogenerator and therefore plays a key role in vibration-based energy harvesting applications.

### 4.1 Equation of Motion in Relative Coordinates

Using the relative displacement formulation, the equation of motion can be expressed as:

$$mx''(t) + cx'(t) + kx(t) = -my''(t) \quad (5)$$

Assuming harmonic base excitation of the form:

$$y(t) = Y\sin(\omega t) \quad (6)$$

the second derivative of the base motion becomes:

$$y''(t) = -\omega^2 Y\sin(\omega t) \quad (7)$$

Substituting Eq. (7) into Eq. (5), the governing equation reduces to:

$$mx''(t) + cx'(t) + kx(t) = m\omega^2 Y\sin(\omega t) \quad (8)$$

### 4.2 Steady-State Harmonic Response

To obtain the steady-state vibration response, a harmonic solution is assumed:

$$x(t) = \Re\{\tilde{X}e^{i\omega t}\} \quad (9)$$

Where  $\tilde{X}$  is the complex amplitude of the response and  $i = \sqrt{-1}$ .

Substituting Eq. (9) into Eq. (8) yields:

$$(-m\omega^2 + ic\omega + k)\tilde{X} = m\omega^2 Y \quad (10)$$

Solving for the complex response amplitude gives:

$$\tilde{X} = m\omega^2 Y / (k - m\omega^2 + ic\omega) \quad (11)$$

### 4.3 Vibration Amplitude and Phase Angle

The steady-state vibration amplitude is obtained by taking the magnitude of the complex response:

$$X = |\tilde{X}| = m\omega^2 Y / \sqrt{(k - m\omega^2)^2 + (c\omega)^2} \quad (12)$$

The phase angle between the excitation and the response is given by:

$$\phi = \tan^{-1}(c\omega/k - m\omega^2) \quad (13)$$

Thus, the steady-state relative displacement can be written as:

$$x(t) = X \sin(\omega t - \phi) \quad (14)$$

### 4.4 Normalized Frequency Response Function

Introducing the natural frequency  $\omega_n = \sqrt{k/m}$  and the damping ratio  $\zeta = c/(2m\omega_n)$  the vibration amplitude ratio can be expressed in nondimensional form as:

$$X/Y = (\omega/\omega_n)^2 / \sqrt{(1 - (\omega/\omega_n)^2)^2 + (2\zeta\omega/\omega_n)^2} \quad (15)$$

Equation (19) clearly illustrates the resonance behavior of the system, where the vibration amplitude reaches its maximum near the natural frequency.

### 4.5 Velocity Response and Energy Interpretation

The relative velocity response is obtained by differentiating Eq. (14):

$$x'(t) = \omega X \cos(\omega t - \phi) \quad (16)$$

Since the electrical output of triboelectric nanogenerators is strongly influenced by the contact–separation velocity, the vibration velocity response plays a crucial role in the energy harvesting mechanism.

The mechanical power dissipated through damping can be expressed as:

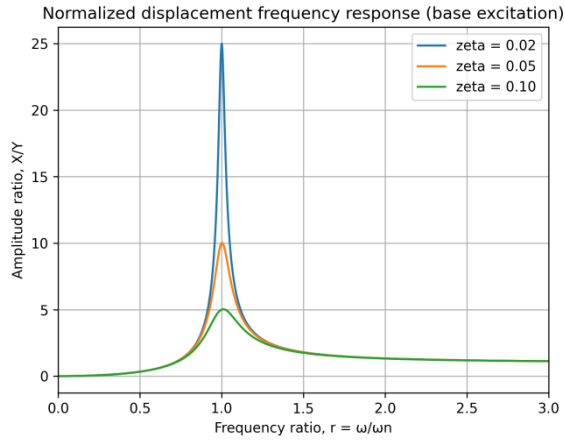
$$P_d = (1/2)c^2\omega^2 X \quad (17)$$

This dissipated mechanical power represents the portion of vibrational energy that can potentially be converted into electrical energy through the triboelectric mechanism, highlighting the importance of vibration amplitude and system tuning in energy harvesting applications.

## 5. Results and Parametric Analysis

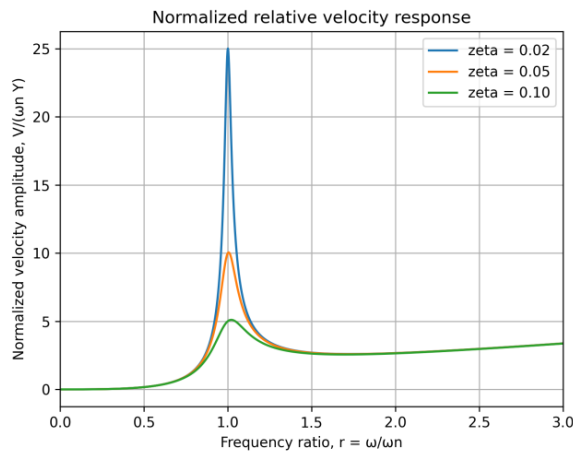
After conducting studies and governing equations, in order to observe numerical effects, 3 zeta numbers (0.02, 0.05, 0.1) were used as default in drawing the graphs. The analytical frequency response is used to investigate the effect of damping on the steady-state relative motion. Figure 2

presents the normalized displacement amplitude ratio  $X/Y$  versus the frequency ratio  $r = \omega/\omega_n$  for different damping ratios. A pronounced resonance region is observed near  $r \approx 1$ . Increasing the damping ratio reduces the peak amplitude and broadens the response, indicating a trade-off between vibration amplification and bandwidth.



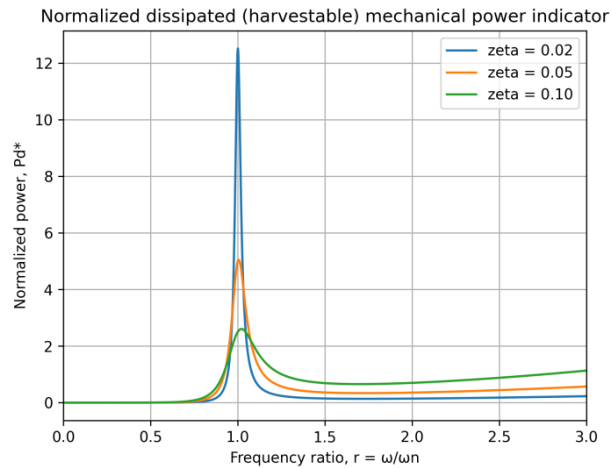
**Figure 2.** Frequency response of the relative displacement for harmonic base excitation: normalized amplitude ratio  $X/Y$  versus frequency ratio  $r = \omega/\omega_n$  for different damping ratios.

Figure 3 shows the normalized relative velocity amplitude, which is directly related to the contact–separation rate in vibration-driven triboelectric nanogenerators. The velocity response increases rapidly near resonance and remains sensitive to damping variations.



**Figure 3.** Normalized relative velocity amplitude  $V/(\omega_n Y)$  versus frequency ratio  $r$  for different damping ratios. This response is directly relevant to contact–separation rate in vibration-driven TENGs.

From an energy perspective, Figure 4 illustrates the normalized dissipated mechanical power associated with viscous damping. This quantity represents the portion of vibrational energy that can potentially be converted into electrical energy. The results indicate that maximum energy conversion potential occurs near resonance, while damping significantly influences both peak power and frequency bandwidth.



**Figure 4.** Normalized dissipated (harvestable) mechanical power  $P_d$  versus frequency ratio  $r$  for different damping ratios. Peaks indicate frequency ranges with higher energy conversion potential.

## 6. Discussion

The analytical results highlight the importance of dynamic tuning in vibration-based triboelectric energy harvesting systems. Operating near resonance significantly enhances vibration amplitude and relative velocity, which are key factors governing triboelectric energy conversion. This behavior is particularly advantageous for low-frequency excitation environments where triboelectric nanogenerators outperform conventional harvesters.

The simplified modeling approach adopted in this study provides clear physical insight into the vibration behavior without relying on complex multiphysics simulations. The framework can be extended in future studies to include nonlinear effects, detailed electromechanical coupling, and experimental validation.

## 7. Conclusions

An analytical dynamic study of a vibration-based energy harvesting system incorporating a triboelectric nanogenerator was presented. The system was modeled as a single-degree-of-freedom mass–spring–damper system subjected to harmonic base excitation. Closed-form solutions for the steady-state response were derived, and the frequency response characteristics were analyzed. The results demonstrate that vibration amplitude, relative velocity, and energy conversion potential are maximized near resonance. The presented framework provides a solid theoretical basis for future numerical and experimental investigations of vibration-driven triboelectric energy harvesters.

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