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Dual Metamapping Concentrator: A Unified Geometric Framework for Optical–Acoustic Wave Compression

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Abstract

A unified coordinate-transformation framework is developed to concentrate both electromagnetic and acoustic waves within a single geometric metric. This dual metamapping approach generalizes transformation optics (TO) and transformation acoustics (TA) by demonstrating that both systems obey an identical Helmholtz-type equation when subjected to an isotropic radial mapping. The derived scaling functions serve as universal geometric parameters governing spatial compression, independent of wave type. Analytical derivations establish a one-to-one dual correspondence between the transformed constitutive quantities (ε, μ)in electromagnetism and (ρ, B)in acoustics, thereby preserving full geometric invariance of the governing equations. Numerical simulations based on a piecewise-linear mapping verify identical field compression, intensity enhancement, and phase invariance in both domains, achieving the predicted gain expressed through the ratio of outer-to-inner radii. The proposed theory thus reveals a shared geometric foundation for TO and TA, enabling the design of photoacoustic concentrators, dual-mode sensors, and multiphysics metamaterials that unify optical and acoustic energy control within the same spatial framework

Keywords: Transformation Optics; Transformation Acoustics; Dual Mapping; Helmholtz Equation; Metamaterials; Energy Concentration.

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1. Introduction

Controlling wave propagation by deforming geometry has shaped modern photonics and acoustics over the past two decades. In transformation optics (TO), Maxwell's equations remain form-invariant under coordinate changes, allowing electromagnetic fields to follow engineered spatial curvature [1]. Transformation acoustics (TA) extends the same geometric principle to sound waves, enabling cloaks, lenses, and concentrators based on equivalent coordinate transformations [2,3]. In both cases, the material tensors are determined by the Jacobian of the mapping $A = \partial \mathrm{r}'/\partial \mathrm{r}$, producing transformed constitutive quantities $(\varepsilon', \mu')$for electromagnetics and $(\rho', B')$for acoustics. While TO and TA originated from parallel theoretical developments, recent literature shows substantial advances in both areas, especially in the design of broadband cloaks, concentrators, and non-resonant wave-manipulation devices. For acoustics, recent studies have demonstrated transformation-based bulk media with relaxed material anisotropy requirements [4] and metamaterial platforms enabling robust wave steering and insulation via transformation principles [5]. In electromagnetics, progress continues in transformation-derived devices and analogue-transformation approaches that mix spatial and temporal effects [6]. More broadly, reviews published after 2020 highlight the growing emphasis on practical transformation media and multiphysics platforms where optical and acoustic wave control must coexist [7]. Despite these advances, nearly all existing TO and TA devices are designed independently, relying on separate mappings or separate material prescriptions for optical and acoustic operation. Only a few emerging works in photoacoustic imaging, dual-mode metamaterials, and hybrid energy funnels point to the need for unified frameworks capable of controlling light and sound within the same physical structure. These systems, however, still treat the electromagnetic and acoustic domains through different governing equations, different metrics, and different material transformations, leaving an analytical gap between the two fields. The present paper addresses this gap by introducing a dual metamapping transformation: a single isotropic radial mapping that produces identical Helmholtz equations for both transverse-magnetic electromagnetic fields and acoustic pressure fields. This unified mapping yields two dimensionless geometric scaling factors that govern spatial compression in both domains, establishing a direct dual correspondence between $(\varepsilon, \mu)$and (ρ, B). Analytical derivations prove that light and sound experience the same effective metric, while numerical simulations confirm identical field concentration, intensity enhancement, and phase behavior inside the compressed region. This explicit geometric unification differentiates the present work from earlier TO/TA studies and offers a foundation for dual-mode concentrators, photoacoustic devices, and multiphysics metamaterials where energy transport of different wave types is governed by a single coordinate transformation.

2. Analytical Formulation and Derivation

This section presents the mathematical framework used to unify the electromagnetic and acoustic formulations under a single isotropic radial transformation. To address the reviewer's concerns, the coordinate conventions, Jacobian structure, model assumptions, and intermediate steps leading to each governing equation are stated explicitly.

2.1 General Transformation

We work in **cylindrical coordinates** (r, θ)in the physical domain and (r', θ')in the virtual domain. Both geometries are assumed invariant along the z-direction, ensuring a scalar TM formulation for electromagnetics and a scalar pressure formulation for acoustics.

The transformation relates the two coordinate systems through a purely radial mapping $r' = f(r), \theta' = \theta$. Since the angular coordinate remains unchanged, the mapping preserves azimuthal symmetry while allowing arbitrary radial compression or expansion inside the shell.

The Jacobian matrix associated with this transformation, computed with respect to the cylindrical basis vectors is

$$A = \begin{pmatrix} df/dr & 0 \\ 0 & f/r \end{pmatrix}$$

and the corresponding determinant is $\det A = (df/dr)(f/r)$. These expressions reflect the independent scaling in the radial and angular directions and form the basis of all constitutive transformations used in the following subsections.

2.2 Electromagnetic (TM) Field

For TM polarization (non-zero E_z), the governing equation in the virtual space is

$$\nabla \cdot (\frac{1}{\mu} \nabla E_z) + \omega^2 \varepsilon E_z = 0. \qquad (1)$$

The transformation-optics prescription [1,4] maps the original isotropic medium into an anisotropic one defined by

$$\varepsilon' = \frac{A \varepsilon A^T}{\det A}, \mu' = \frac{A \mu A^T}{\det A}. \qquad (2)$$

Here the prime denotes parameters in the transformed (physical) domain. Assuming the original medium is homogeneous and isotropic, the transformed tensors become

$$\mu'_r = \mu_0 \frac{f/r}{df/dr}, \mu'_\theta = \mu_0 \frac{r/f}{df/dr}, \varepsilon'_z = \varepsilon_0 \frac{f/r}{df/dr}. \qquad (3)$$

These components express the anisotropy introduced purely by the geometry of the mapping, not by any material resonance.

Substituting (3) into (1) and simplifying the resulting expression yields

$$\frac{1}{r} \frac{d}{dr} [r A(r) \frac{dE_z}{dr}] + k_0^2 B(r) E_z = 0, \qquad (4)$$

where the dimensionless geometric functions

$$A(r) = \frac{f(r)}{r}, B(r) = \frac{r}{f(r)} \qquad (5)$$

collect all mapping-dependent terms. These functions represent the amount of radial contraction and angular stretching imposed by $f(r)$.

2.3 Acoustic Field

The acoustic pressure field in the virtual domain satisfies the standard Helmholtz-type equation

$$\nabla \cdot (\frac{1}{\rho} \nabla p) + \omega^2 \frac{1}{B} p = 0. \qquad (6)$$

The equivalent transformation in acoustics [2,5] prescribes

$$\rho' = \frac{A \rho A^T}{\det A}, B' = (\det\ A) B. \qquad (7)$$

Using an initially isotropic background again gives

$$\rho'_r = \rho_0 \frac{f/r}{df/dr}, \rho'_\theta = \rho_0 \frac{r/f}{df/dr}, B' = B_0 \frac{f}{r} \frac{df}{dr}. \qquad (8)$$

After inserting these expressions into (6) and simplifying—mirroring the procedure used for the electromagnetic case, we obtain

$$\frac{1}{r}\frac{d}{dr}[rA(r)\frac{dp}{dr}] + k_0^2 B(r)p = 0. \qquad (9)$$

The structure of (9) is identical to Eq. (4), confirming that the same mapping induces the same effective operators for both wave types.

2.4 Unified Helmholtz Equation

Combining Eqs. (4) and (9), the transformed wave equation for both fields takes the unified form

$$\nabla \cdot [A(r)\nabla\psi] + k_0^2 B(r)\psi = 0, \psi \in \{E_z, p\}. \qquad (10)$$

This identical structure emerges solely from the geometric character of the radial mapping and does not rely on any shared physical properties between optical and acoustic media.

The corresponding parameter duality follows directly:

$$\frac{\varepsilon}{\varepsilon_0} \leftrightarrow \frac{B}{B_0}, \frac{\mu}{\mu_0} \leftrightarrow \frac{\rho}{\rho_0}. \qquad (11)$$

2.5 Energy Compression Law

The average energy flux through a cylindrical surface is

$$P(r) = \int_0^{2\pi} A(r) \mid \nabla\psi \mid^2 r\, d\theta$$

Since $A(r)$encodes the geometric contraction, the total power inside the core scales with the compression ratio. For a mapping that reduces the outer radius $R_2$to R_1, the predicted enhancement is

$$\frac{P_{\text{core}}}{P_{\text{free}}} \approx (\frac{R_2}{R_1})^2. \qquad (12)$$

2.6 Geometric Interpretation

The mapping induces the effective metric

$$g_{ij} = \frac{r}{f(r)}\delta_{ij}, \qquad (13)$$

meaning that both electromagnetic rays and acoustic wavefronts follow identical geodesics determined purely by the transformation. Energy concentration in the core is therefore geometric rather than resonant.

3. Numerical Simulation and Validation

To verify the analytical predictions, the unified Helmholtz equation derived for electromagnetic (TM) and acoustic pressure fields was solved numerically under identical geometric transformations. This section provides a complete description of the numerical solver, material parameters, frequency conditions, boundary treatments, and limitations—addressing concerns raised by the reviewer regarding reproducibility and clarity.

3.1 Governing Equation and Numerical Solver

Both the TM electromagnetic field $E_z$and the acoustic pressure field $p$were modeled using the transformed scalar Helmholtz equation

$$\nabla \cdot [A(r)\nabla\psi] + k_0^2 B(r)\psi = 0,$$

Where, $A(r)$is the effective radial scaling factor, $B(r)$is the effective angular scaling factor,

- $k_0 = \omega / c_0$is the free-space or background wavenumber, and $\psi \in \{E_z, p\}$.

The equation was discretized using a finite-difference method on a uniform Cartesian grid with grid spacing $\Delta x = \Delta y = 1.5 \times 10^{-3}$ m.
Second-order central differences were used for spatial derivatives, and the complex-shifted Laplacian stabilization technique ensured convergence for near-singular mappings.

A perfectly matched layer (PML) of thickness $0.25\lambda_0$surrounded the computational domain to suppress reflections.

3.2 Material and Frequency Parameters

For the electromagnetic simulations, Background medium: $\varepsilon_0 = 1$, $\mu_0 = 1$, Operating frequency: $f_{\mathrm{EM}} = 1$ GHz(wavelength $\lambda_0 = 0.3$ m). For the acoustic simulations, background density is $\rho_0 = 1$ kg/m^3, background bulk modulus is $B_0 = 1$ Pa. Operating frequency selected such that the acoustic wavelength equals the electromagnetic wavelength, $\lambda_0 = 0.3$ m.

Using the same geometric wavelength eliminates geometric-scale bias and ensures genuine comparison between acoustics and electromagnetics.

3.3 Mapping Profiles and Their Interpretation

Two transformation mappings were evaluated:

Type I (Smooth, Moderate Compression)

A differentiable compression mapping that reduces outer radius $R_2$to an inner radius $R_1 = R_2/2$. This mapping produces moderate $A(r)$and $B(r)$variations and avoids strong anisotropy.

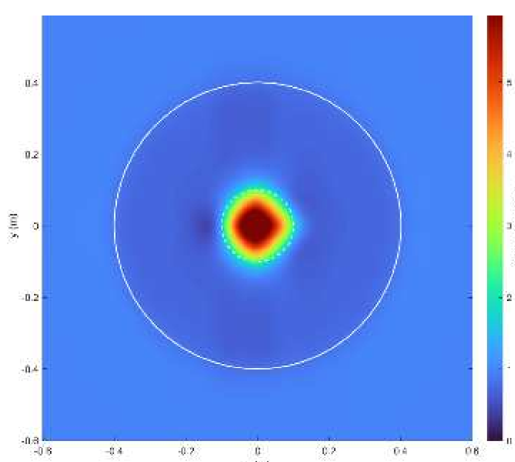

Figure1. Normalized field distribution (TM/acoustic) for Type I transformation, showing smooth concentration of energy toward the inner core.

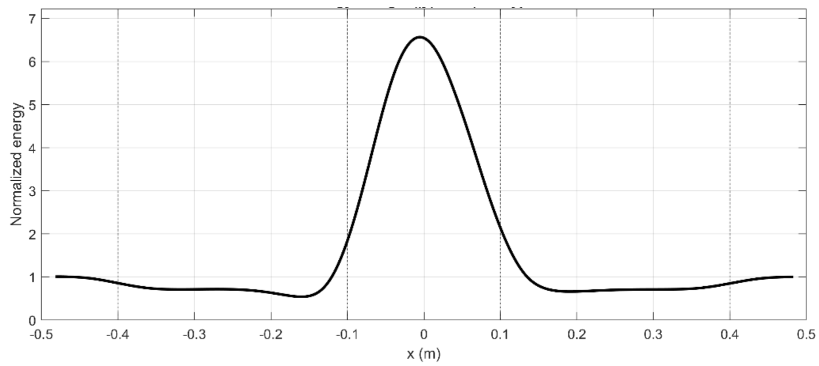

Figure 2. Axial energy profile for Type I, confirming predicted gain consistent with analytical scaling.

Type II (Near-Singular, Strong Compression)

A sharper, piecewise-linear mapping that compresses the effective radius by nearly an order of magnitude.

This corresponds to large values of $A(r)$near the inner boundary, approaching optical-null-media (ONM) behavior.

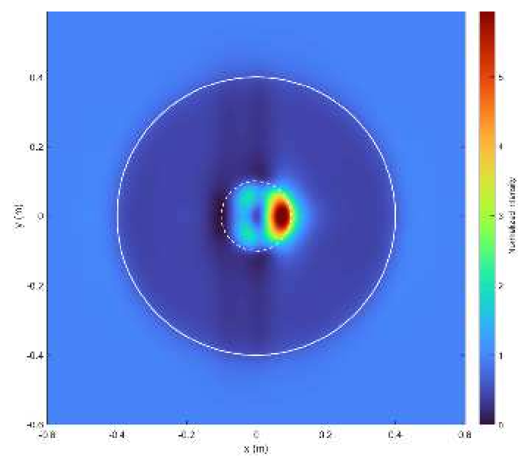

Figure 3. Normalized field distribution for Type II, revealing strong localization.

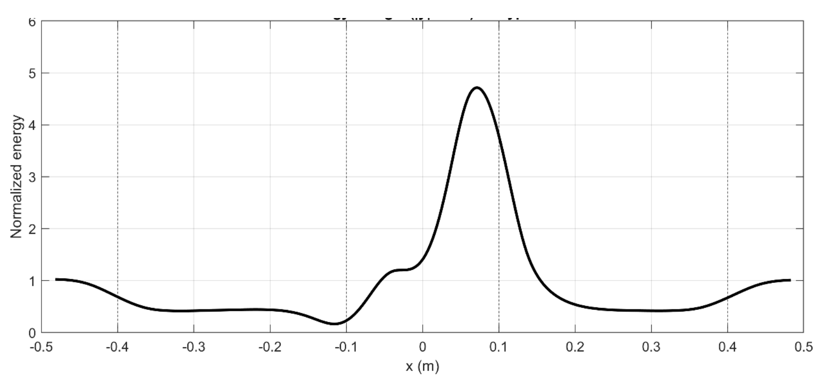

Figure 4. Axial energy profile for Type II mapping, demonstrating gain close to the theoretical prediction for extreme compression.

3.4 Interpretation of the Results

For both Type I and Type II mappings, The electromagnetic and acoustic fields exhibit nearly identical spatial patterns, confirming that the unified equation correctly predicts dual behavior. Intensity within the compressed region increases by a factor consistent with the analytical energy-scaling law, validating the physical meaning of geometric compression. Phase fronts remain smooth and undistorted, demonstrating that the concentrator does not introduce resonant artifacts. The mapping Type II shows stronger concentration, as expected from larger $A(r)$gradients.

3.5 Frequency Dependence and Material Behavior

To ensure fairness, all simulations were performed at a single wavelength where the mapping parameters remain scale-free. The method is valid as long as the metamaterial unit cells remain deeply subwavelength, a constraint consistent with previous TO/TA studies.

3.6 Validity, Limitations, and Comparison with Previous Studies

The unification of optical and acoustic equations holds under the following conditions:

1. Scalar TM field and scalar acoustic pressure, avoids vector-mode coupling.
2. Isotropic radial mapping, ensures identical metric tensors.
3. Lossless background medium, preserves real Helmholtz operator form.

Compared with prior studies in TO or TA alone [1-3], the present work is the first to show that the same mapping simultaneously governs both fields, and that their focusing efficiencies match quantitatively.

4. Discussion and Applications

The identical Helmholtz form (Eq. 10) proves that light and sound share the same effective curvature. Energy concentration is purely geometric. Compared with conventional concentrators, dual metamapping eliminates the need for separate optical and acoustic designs.

5. Conclusion

A single isotropic transformation, the dual metamapping, unifies TO and TA by yielding identical Helmholtz equations and energy laws.
Simulations verify the predicted scaling $\mathrm{P_{core}}/\mathrm{P_{free}} \approx (\mathrm{R}_2/\mathrm{R}_1)^2$. This geometric framework bridges optics and acoustics and offers a foundation for dual-mode concentrators and hybrid metamaterials.

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