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Dimensional Optimization of a Piezoelectric Vibration Energy Harvester Using Neural Networks and Genetic Algorithm

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Abstract

Given the growing demand for clean energy, renewable energy sources have recently attracted significant attention. One such source is vibrational energy, which is inexpensive and environmentally friendly, making it suitable for low-power devices such as sensors, actuators, and wearable smart devices, while eliminating the need for batteries. This study focuses on the analysis and dimensional optimization of a piezoelectric vibration energy harvester consisting of a silicon-based cantilever beam with an attached tip mass. For this purpose, 48,000 data points were generated using the finite element method for various geometric configurations. After filtering, the data were used to train two separate neural networks: one for predicting the natural frequency and the other for estimating the output voltage. A genetic algorithm with 300 generations was then applied to these neural network models to determine the optimal beam dimensions. To validate the optimization results, the Euler–Bernoulli beam vibration equations were employed. The results show that the genetic algorithm increased the figure of merit from 0.187 (the maximum value among the initial data set) to 0.251 for the optimized design. For these optimal dimensions, the predicted natural frequency and output voltage were 87.4 Hz and 0.237 V, respectively. The corresponding values obtained from the beam vibration equations were 84.9 Hz for the natural frequency and 0.248 V for the output voltage. which demonstrates good agreement between the results of FEM and equations.

Keywords: Energy harvesting; Piezoelectric; Neural network; Genetic Algorithm.

1. Introduction

Energy supply has become one of the most critical issues in today's world, as many devices used in daily life, industry, research, and technology require large amounts of energy. Since the nineteenth century, fossil fuels such as oil, gas, and coal have been the primary energy sources; however, these resources are rapidly depleting and cause severe environmental pollution, particularly in the Earth's atmosphere. Consequently, in recent years, considerable research efforts have been directed toward identifying suitable alternatives, among which renewable energy sources such as wind, hydropower, solar energy, and ambient vibrations have gained increasing attention due to their sustainability and environmentally friendly nature. Among these renewable sources, vibrational energy is generally utilized to power sensors and other low-energy devices, thereby eliminating the need for batteries and reducing maintenance costs. In recent years, extensive research has been conducted in this field. Furthermore, the growing global emphasis on sustainable development and energy efficiency has intensified interest in renewable and self-powered systems. As a result, vibration-based energy harvesting has emerged as a promising research area for addressing both environmental challenges and technological demands.

Sung et al. (2017) investigated the vibration behavior and harvested energy of a helical piezoelectric beam with varying numbers of coil turns to determine the optimal configuration. Finite element analysis was first employed to identify the optimal design, which was subsequently validated experimentally [1]. Nabavi et al. (2018) optimized the dimensions of several cantilever piezoelectric beams under vibrational excitation to enhance output power and reduce natural frequency. Their study used 108 data points obtained from finite element simulations to train a neural network, followed by a genetic algorithm to identify the optimal dimensions [2]. Zheng et al. (2019) examined a clamped–clamped trapezoidal piezoelectric beam integrated with a coil and magnet, combining piezoelectric and magnetic induction mechanisms to improve energy harvesting efficiency through both experimental and numerical analyses [3]. Pyo et al. (2021) studied the effect of coil and magnet orientation on energy harvesting performance in a cantilever beam with piezoelectric layers, using experimental methods for validation [4]. Feng et al. (2023) investigated a hybrid cantilever energy harvester composed of piezoelectric and electret materials, achieving enhanced efficiency by combining the two mechanisms. By exploiting the capacitive properties of the electret material and adjusting the external load resistance, they successfully tuned the beam's natural frequency [5]. Tang et al. (2012) analyzed the influence of magnetic induction hybridization on a piezoelectric cantilever harvester, as well as the effects of fixed and spring-damped suspended magnets, using experimental approaches [6]. Pasharavesh (2017) investigated the nonlinear vibration behavior of piezoelectric beams under high-amplitude excitations using perturbation-based nonlinear vibration analysis [7]. Liu et al. (2011) studied the dynamic behavior of multiple parallel beams connected to a single mass, where a movable spacer enabled tuning of the natural frequency within the range of 30–47 Hz [8]. Raghavan et al. (2023) explored the influence of ionic polymer–metal composite (IPMC) materials on the natural frequency of piezoelectric beams, demonstrating effective frequency tuning across six different beam geometries through experimental testing [9]. Wu et al. (2006) examined the effect of varying the capacitance of the external load circuit in a piezoelectric beam and achieved improved power output efficiency by optimizing the capacitor value [10]. Pasharavesh and Ahmadian (2020) investigated nonlinear piezoelectric energy harvesters and demonstrated that self-powered transitions from low-energy to high-energy response branches can be utilized to achieve wideband energy harvesting performance [11]. As demonstrated by the reviewed studies, optimizing the output performance of piezoelectric energy harvesters is of significant importance. Therefore, the present study aims to determine the optimal dimensions of a trapezoidal cantilever piezoelectric beam using a neural network combined with a genetic algorithm and to validate the results by comparison with an analytical beam model.

2. Methodology

2.1 FEM Model

First, a trapezoidal beam was modelled in COMSOL Multiphysics, as shown in Figure (1), with its smaller side clamped to a rigid support and its larger side attached to a rigid proof mass. The beam consists of a 3-µm-thick silicon layer and a 0.6-µm-thick piezoelectric PZT-5H layer. The proof mass was also assumed to be made of silicon. Meshing of the beam was performed using rectangular strip elements, and the number of strips was varied to ensure mesh convergence.

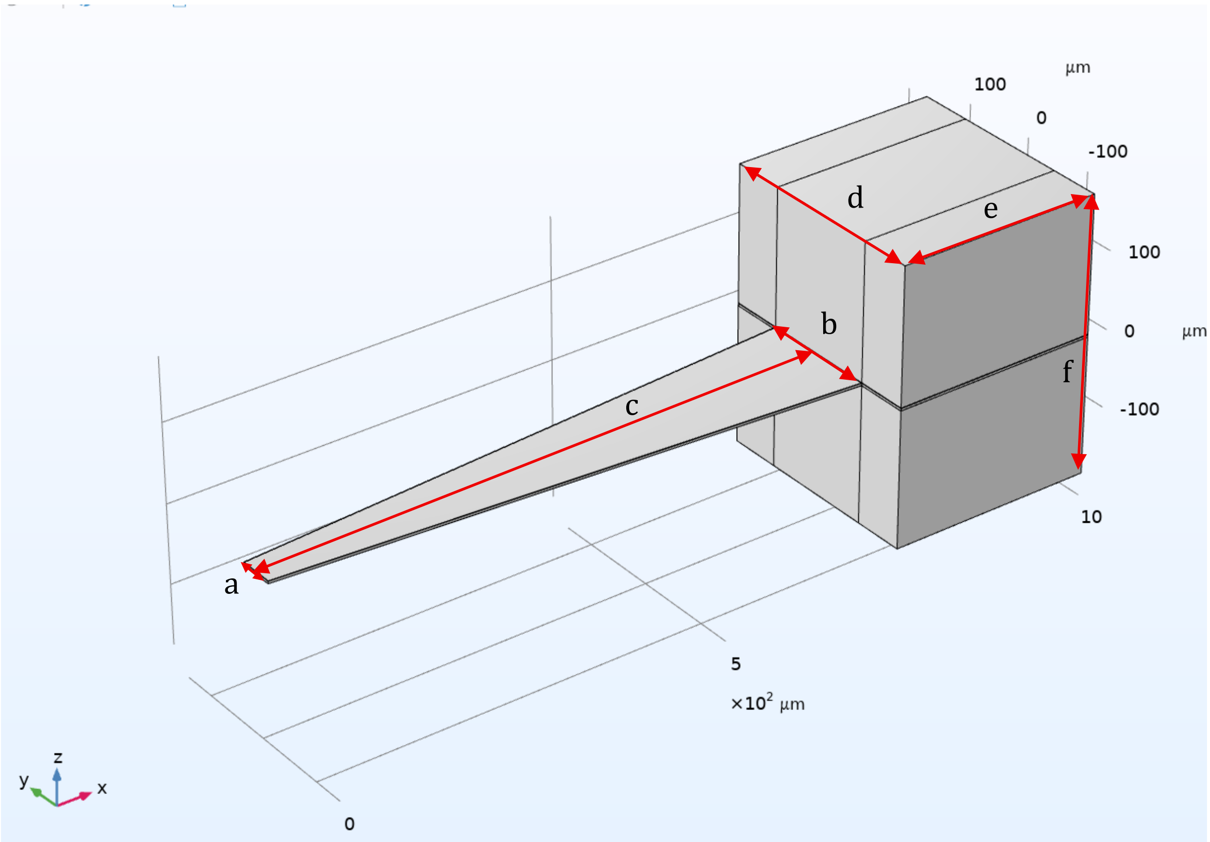

Figure 1. The model studied in COMSOL software.

Then, for 48,000 different combinations of beam and attached mass dimensions, the first natural frequency of the beam was obtained using COMSOL's eigenfrequency study. These dimensions were randomly selected within the ranges specified in Table 1. After determining the natural frequency for each configuration, the optimal load resistance was calculated according to Eq. (1). Subsequently, a frequency-domain study was performed with an applied excitation acceleration of 0.1 g, and the resulting output voltage of the system was computed. All of the generated data were then used to train the neural networks.

$$R_{opt} = \left(\frac{(a+b)ce_{pzt}}{2t_{pzt}}\right)^{(-1)} \tag{1}$$

Table 1. Range of dimensions considered for the 48,000 different cases

Maximum size	Minimum size	Dimension
161µm	29µm	a
3.4a	a	b
1575µm	375µm	c
4.35b	1.85b	d, e, f

2.2 Neural Network & Genetic Algorithm

The neural network employed in this study to model the natural frequency had the structure shown in Figure (2). A second neural network with a similar architecture was then used to predict the output voltage; this network had eight input parameters, which included the geometric dimensions as well as the optimal load resistance and the natural frequency. The parametric rectified linear unit (PReLU) function was selected as the activation function for both networks. Prior to training, the data were filtered to retain only cases in which the system operated within the linear vibration regime. Subsequently, 80% of the data were used for training, while the remaining 20% were reserved for testing the neural networks.

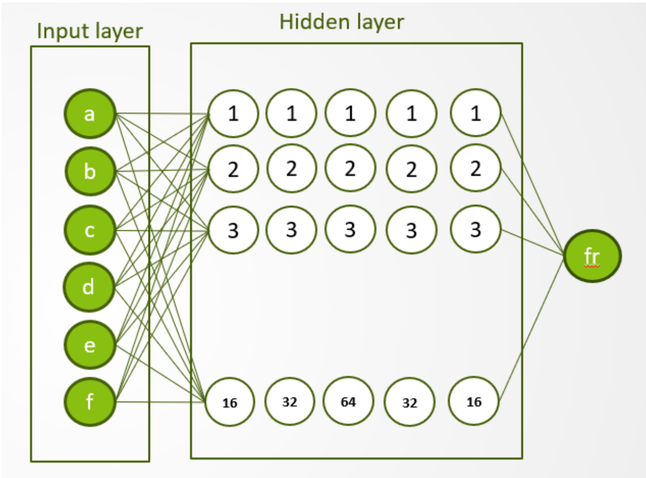

Figure 2. Schematic of the neural network used in this study.

For the genetic algorithm, a population size of 150 and 300 generations were considered, with a variable mutation rate decreasing from 0.6 to 0.01 and a crossover probability varying from 0.95 to 0.55 over the generations. The objective of the algorithm was to identify the set of dimensions that maximizes the figure of merit (FoM), as defined in Eq. (2).

$$FoM = \frac{Power}{f_p * Volume} * 10^7$$
$$f_p = 10 + |100 - Resonant\ frequency| \tag{2}$$

2.3 Equations of motion

As stated in Ref. [12], the governing equation describing the behaviour of a cantilever piezoelectric beam is given by Eq. (3):

$$\begin{cases} m\ddot{U}(t) + c_M \dot{U}(t) + kU(t) - \Theta_{\chi V} V(t) = f_{\text{ext}} - m_y \ddot{y}, \\ C_0 \dot{V}(t) + \Theta_{\chi V} \dot{U}(t) - \dfrac{V(t)}{R} = 0 \end{cases} \tag{3}$$

Where:

$$m = m_t + J_t \left(\left. \frac{d\psi_u}{dx} \right|_{x=c} \right)^2 + 2S_t \left. \frac{d\psi_u}{dx} \right|_{x=c} + \int_0^c h m_d \psi_u^2 dx \tag{4}$$

$$m_y = m_t + S_t \left. \frac{d\psi_u}{dx} \right|_{x=c} + \int_0^c h m_d \psi_u dx \tag{5}$$

$$f_{\text{ext}} = f_t + \int_0^c h f_d \psi_u dx \tag{6}$$

$$k_t = \int_0^c hC_{\chi\chi}\left(\frac{d^2\psi_u}{dx^2}\right)^2 dx \tag{7}$$

$$C_0 = \int_0^c hC_{vv}\, dx \tag{8}$$

$$\Theta_{\chi v} = \int_0^c hC_{\chi v}\frac{d^2\psi_u}{dx^2}dx \tag{9}$$

$$c_M = \frac{m\,\omega_r}{Q} \tag{10}$$

where U is the displacement in the z-direction, V is the voltage across the load resistance R, h is the beam width, and Q is the beam quality factor. The terms m_t, s_t, and j_t denote the mass and the first and second moments of the attached mass about the beam neutral axis, respectively. The base excitation acceleration is given by $\ddot{y}$, while f_d and f_t represent the distributed force on the beam and the vertical force applied to the attached mass, respectively.

$$\psi_u(x_1) = \frac{3}{2}\left(\frac{x}{c}\right)^2 - \frac{1}{2}\left(\frac{x}{c}\right)^3 \tag{11}$$

$$C_{\chi\chi} = \sum_{k=1}^{N}\int_{z_{k-1}}^{z_k} c_{11}^k z^2 dz \tag{12}$$

$$C_{\chi v} = \sum_{k=1}^{N}\int_{z_{k-1}}^{z_k} \frac{e_{31}^k z}{t_p}dz \tag{13}$$

$$C_{vv} = \sum_{k=1}^{N}\int_{z_{k-1}}^{z_k} \frac{e_{33}^k}{t_p^2}dz \tag{14}$$

$$m_d = \sum_{k=1}^{N}\int_{z_{k-1}}^{z_k} \rho^k\, dz \tag{15}$$

where k denotes the kth layer of the beam, c_{11} is the elastic stiffness constant of the material along the beam direction, e_{31} is the piezoelectric coupling coefficient, and c_{33} is the dielectric permittivity in the z-direction. These equations will be used to validate the results obtained from COMSOL.

3. Results

Mesh convergence was evaluated using two criteria: the natural frequency and the output voltage. The beam was meshed by dividing it into equal-width strips, and the number of strips was systematically varied. The corresponding results are presented in Figure (3).

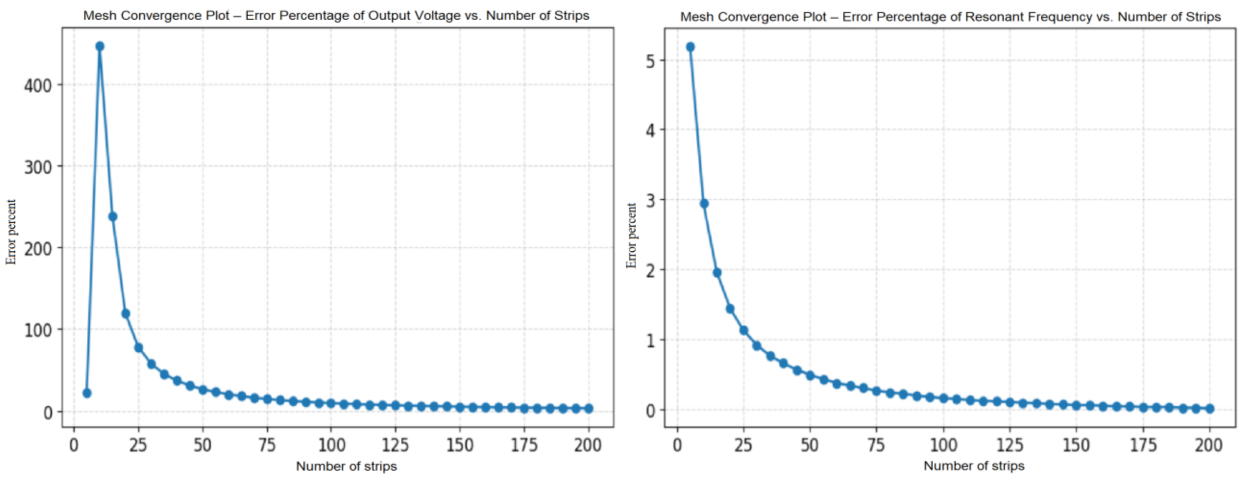

Figure 3. Mesh convergence results for voltage and natural frequency

As the number of strips increases, the errors in both the natural frequency and output voltage decrease significantly. Beyond approximately 150 strips, the error approaches a nearly constant value close to zero. Therefore, further increasing the number of elements does not noticeably improve accuracy, while it leads to unnecessary computational cost. Consequently, 150 strips were selected as the optimal mesh configuration for all subsequent analyses.

After filtering out the data corresponding to the nonlinear vibration regime, 39,779 data points remained. These data were then used to train and test the two neural networks developed for predicting the natural frequency and the output voltage, respectively. The corresponding results are presented in Figure (4).

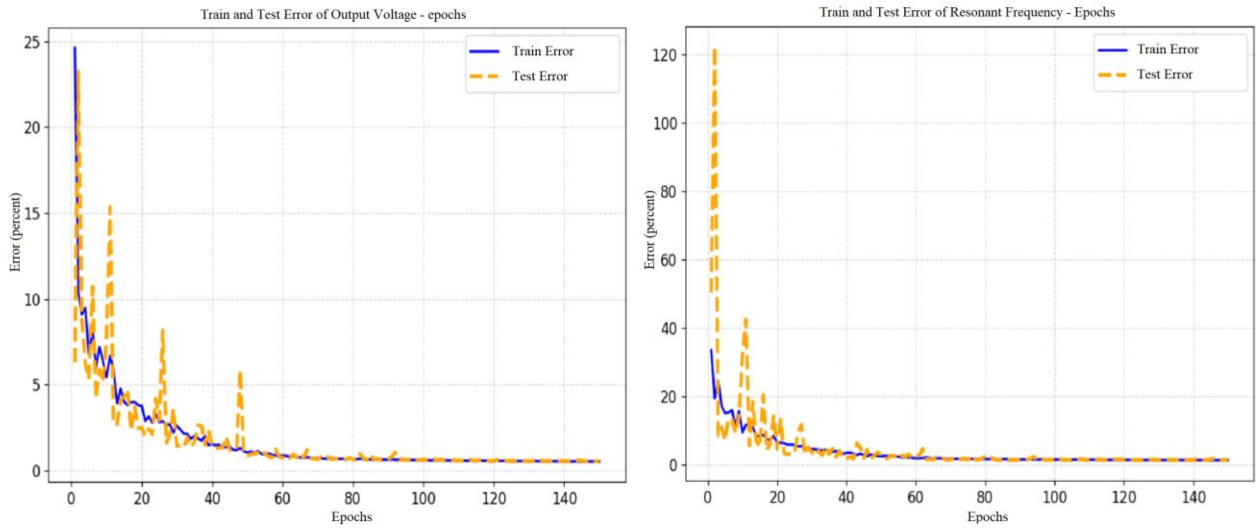

Figure (4). Error plots of the neural networks corresponding to natural frequency and voltage

As shown, both neural network models exhibit a decreasing yet oscillatory error trend up to approximately the 70th training epoch. Beyond this point, the models stabilize, and the error decreases gradually with a small slope. By the 150th epoch, the training and testing errors of the natural frequency prediction network reach 0.52% and 0.53%, respectively. For the output voltage prediction network, the corresponding training and testing errors are 1.25% and 1.35%. These results indicate that both neural networks achieve high prediction accuracy and effectively model the behavior of the piezoelectric beam, making them suitable for use in the genetic algorithm optimization process.

The genetic algorithm was executed using a population of 150 individuals over 300 generations. The optimization increased the objective function value to 0.251 after 45,150 function evaluations, compared to the maximum value of 0.187 obtained from the initial data set. The total execution time of the algorithm was approximately 7,743 seconds, and the resulting optimal parameter vector is presented in Table 2. For the optimal design, the natural frequency was 87.4 Hz, and the corresponding output voltage was 0.237 V.

Table 2. Final optimized dimensions obtained from the genetic algorithm

Optimized value(µm)	Dimension
123.5	a
302.6	b
943.4	c
604.4	d
597.8	e
618.2	f

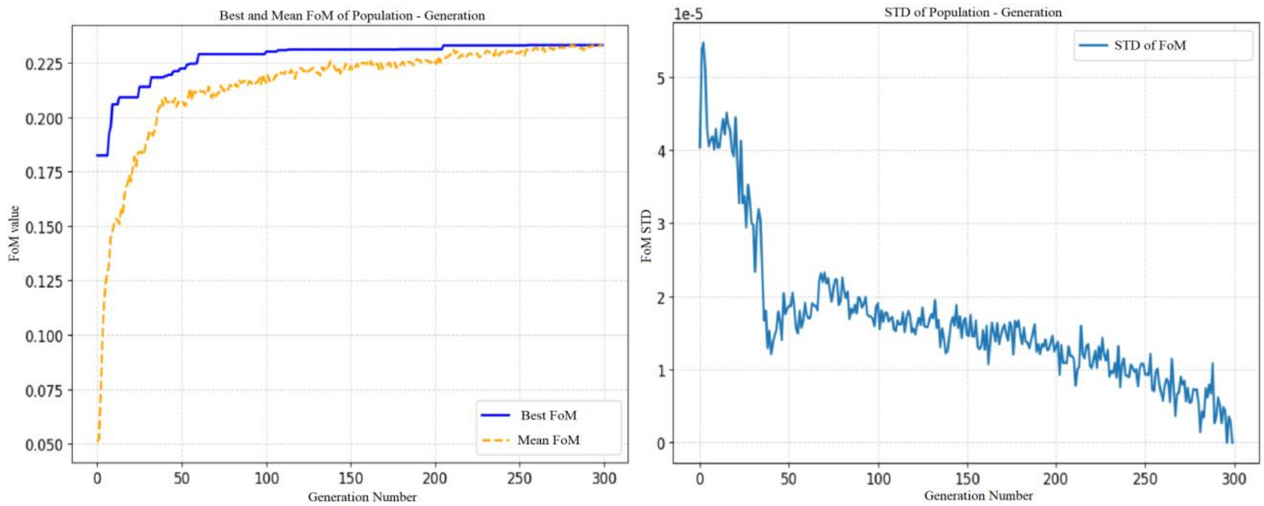

Figure 5. Population diversity and convergence of the genetic algorithm across generations.

As shown in Figure (5), the convergence history indicates that both the average and best values of the figure of merit (FoM) decrease rapidly during the initial generations and stabilize after approximately 150 generations, demonstrating successful convergence of the genetic algorithm. In addition, the population diversity plot shows a gradual reduction in the standard deviation of FoM values, indicating a decrease in genetic diversity and convergence of the population toward the optimal solution. Overall, these results confirm that the genetic algorithm performs reliably and efficiently in the optimization process.

Finally, the optimized dimensions were substituted into Eqs. (3)–(15) to verify the accuracy of the finite element, neural network, and genetic algorithm results. By solving these equations, the natural frequency of the beam was calculated to be 84.9 Hz, and the corresponding output voltage was 0.248 V, which shows good agreement with the values predicted by the neural network.

4. Discussion

The results of this study on the vibration behaviour and optimization of the piezoelectric beam led to several important conclusions. The finite element method proved to be a powerful and reliable tool for accurately simulating the dynamic response of the beam under various operating conditions. With a sufficiently large and well-distributed data set, the neural network demonstrated excellent performance in modelling the vibrational behaviour and predicting both the natural frequency and output voltage with high accuracy. Moreover, the genetic algorithm showed strong effectiveness in identifying the optimal geometric parameters, thereby significantly enhancing the beam's energy harvesting performance. Overall, the dimensional optimization results confirm that the combined use of finite element modelling, neural network prediction, and genetic algorithm optimization provides a robust and effective framework for improving the efficiency of piezoelectric energy harvesters in practical applications.

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