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A Novel Wideband DOA Estimation Method via Decorrelation Approach for Coherent Signals

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Abstract

In this paper, we proposed a new DOA estimation method for wideband coherent signals. First, the signal reaching on the array is broken down into some narrowband signals, and the covariance matrix is calculated at each frequency bin. Then, a Toeplitz matrix reconstruction approach is formed to reduce the correlation between the signals. Based on this matrix, the focusing matrix is calculated and the signal information is transmitted from different narrow-frequency bins to the central frequency. Finally, the DOA estimation is done by solving a cost function in the form of narrowband processing. This method does not require any initial DOA estimation and also does not need any repeating process. The simulation results show the superior performance of the proposed method compared to the other popular methods especially in the cases of coherent scenarios, low signal to noise ratio (SNR), and spatially-closed sources. This method has great potential for practical applications in radar, sonar, and wireless communication systems.

Keywords: Direction of arrival (DOA); Singular Value Decomposition (SVD); WAVES; Signal Subspace Focusing (SSF).

1. Introduction

Direction of Arrival (DOA) estimation is a very important issue in the field of array processing, especially in radar, sonar, satellite, and mobile communications [1-6]. Depending on the nature of the signal and the environment in which the signal is released, array signal processing can include narrow-band and wideband signals. By considering the way of using the information of covariance matrix of different frequency bins, wideband DOA estimation methods are categorized into two general categories of incoherent signal subspace methods (ISSM) [7-8] and coherent signal subspace (CSSM) methods [9]. In ISSM methods, by using narrowband processing approaches, DOA estimation is performed in each frequency bin, separately, and then, the results are interpolated.

In the CSSM methods, first, sample covariance matrices (SCMs) are estimated at several frequency bins, and then, by generating a focusing matrix, SCMs of various frequency bins coherently transmit to the prefixed focusing frequency. In this case, the steering vector of a source in all frequency bins can be identical and consequently can be converted into a single form. Finally, with the averaged transmitted matrices, we can apply narrowband techniques directly over a single covariance matrix.

Incoherent methods work well in optimal conditions. However, in situations where SNR is low or in the presence of non-uniform noise, and even when the directions are spatially close, these methods may suffer from serious performance degradation [10].

In another method, known as the weighted average of signal subspaces (WAVES) [11], the concept of near optimal data-adaptive statistics is combined with an enhanced design of focusing matrix to allow for a strong statistical analysis of the wideband signal. In this method, a pseudo-data matrix is firstly produced by weighted subspace fitting (WSF) method [12] and then, the noise subspaces at the central frequency is estimated from the pseudo-data matrix built with SCM eigenvectors, using SVD approach. This method uses a focusing matrix, and SCM is passed through a filter before it is drawn up. Thus, by applying a weighting matrix, the contribution of signal subspace increases in the method.

One of the important challenges of coherent methods is the need to perform the initial DOA estimate of sources, which is an unknown parameter. To do so, first, the angle of incidence can be calculated approximately using incoherent methods, and then, the focusing matrix can be formed. In order to solve the problem of preliminary estimation of angles in the coherent methods, a method called test of orthogonality of projected subspaces (TOPS) is introduced in [13]. In the TOPS method, the focusing matrix is not used to generate a covariance matrix. Rather, this method uses Rotational Subspace Signal (RSS) of focusing matrix to perform the orthogonality test between the signal and noise subspaces of multiple frequency components. For each DOA assumption, this measure operates at the correct angles and otherwise, this interaction will not be achieved. This method has high computational complexity.

The referred methods perform well when the signals are uncorrelated or partially correlated. But the efficiency of these methods decreases in the case of coherent (fully correlated) signals. In the narrowband signal subspace methods, various methods, such as spatial smoothing (SS) [14-16] and Toeplitz matrix reconstruction methods [17-19], have been used to reduce the correlation of signals and to carry out the de-correlation process. One problem with the subspace-based methods is that they require initial knowledge of the source numbers. In various papers, the information theoretic-based methods, e.g., AIC [20-21], MDL [21-22], and their variants [23-25], have been proposed to overcome the mentioned problem. Most of these methods do not show the optimal efficiency at low SNRs and in the case of correlation between resources. Moreover, In the case of wideband signals, the direct use of narrowband methods, which resolve coherent signals, is not optimal, at all. In this paper, a new DOA estimation method for wideband coherent sources is presented. This method is based on the joint diagonalization structure of the full set of Toeplitz matrices and can be considered as an extension of the references [26-27] in the wideband mode. This method does not require any initial DOA, a priori knowledge on the source number and any repeating process.

The rest of the paper is organized as follows. The signal model is presented in Section 2. In Section 3, the idea of using Toeplitz matrix in the process of de-correlation of wideband signals and also, the process of calculating the focusing matrix are described. In Section 4, simulation results are presented. Finally, the conclusion is given in Section 5.

2. Signal model

We assume that there are D far-field source signals impinging on a uniform linear array (ULA) of 2M + 1 sensors spaced by a half-wavelength corresponding to the highest frequency used in the processing. Assuming that all sources are wideband signals with specified bandwidths, and the frequencies are the same, the output of the K-point discrete Fourier transform (DFT) module can be modelled as:

$$x(f_k) = A(f_k, \theta) s(f_k) + n(f_k), k \in [K] \tag{1}$$

where $x(f_k)$, $s(f_k)$ and $n(f_k)$ are the received data, source signals, and additive noise in the frequency domain, respectively. It is assumed that the noise is a white Gaussian process with zero mean and covariance of σ^2. $A(f_k.\theta) = [a_1(f_k.\theta_1) \quad \dots \quad a_D(f_k.\theta_D)]^{\mathrm{T}}$ denotes the array manifold matrix with the pth steering vector being $\mathrm{a}(\mathrm{f_k}, \theta_\mathrm{i}) = \left[\mathrm{e}^{\mathrm{j}\frac{2\pi \mathrm{f_k} \mathrm{Mdsin}\theta_\mathrm{i}}{\mathrm{c}}}, 1, \dots, \mathrm{e}^{-\mathrm{j}\frac{2\pi \mathrm{f_k} \mathrm{dMsin}\theta_\mathrm{i}}{\mathrm{c}}}\right]^{\mathrm{T}}; \mathrm{i} = 1,2, \dots, \mathrm{D}$, where $j = \sqrt{-1}$, c is the propagation speed and d is the inter-element spacing. The structure of the steering vector is modelled by considering the middle element in the array as a reference point with a zero phase. The received signals can be uncorrelated, partially correlated or fully correlated (coherent). In our subsequent discussions, we assume that P signals are mutually coherent while the others are uncorrelated and independent of the first P signals. Taking the first signal $S_1(f)$ as reference, the pth coherent signal becomes:

$$S_p(f_k) = \alpha_p \, e^{-j2\pi f_k \tau_p} S_1(f_k), \ p \in [P] \tag{2}$$

Where α_p is the amplitude fading factor and τ_p is the relative delay between the first source and the multipath component. Thus, the signals received by the mth sensor can be expressed as:

$$X_m(f_k) = S_1(f_k) \sum_{\mathrm{p}=1}^{P} \alpha_p e^{-j2\pi f_k \tau_p} e^{-j2\pi f_k \frac{mdsin\theta_p}{c}} + \sum_{\mathrm{d}=\mathrm{P}+1}^{D} S_d(f_k) e^{-j2\pi f_k \frac{mdsin\theta_d}{c}} + N_m(f_k) \tag{3}$$

The array covariance matrix at the frequency f_k is modeled as follows:

$$R(f_k) = A(f_k, \theta) R_s(f_k) A(f_k, \theta)^H + R_n(f_k) \tag{4}$$

in which $R_s(f_k) = E[S(f_k)S^H(f_k)]$ is the source covariance matrix and $R_n(f_k) = E[n(f_k)n^H(f_k)]$ is the (2M+1×2M+1) noise covariance matrix. The symbols $\mathrm{E}[.]$ and $(.)^H$ denote the statistical expectation and the Hermitian transpose, respectively. In the case of uncorrelated source and with the assumption D < M, the ranks of the matrices $R(f_k)$ and $A(f_k, \theta)$, for all frequencies and directions, are equal to D. In practice, the exact array covariance matrix R is unavailable, thus, its sample estimate $\hat{R}_{f_k} = \frac{1}{N}\sum_{i=1}^{N} X(f_k.i) X^H(f_k.i)$, is used, where N is the number of snapshots. The eigenvalue-decomposition (EVD) of $\hat{R}_k$ yields:

$$\hat{R}_{f_k} = \frac{1}{N} \sum_{i=1}^{N} X(f_k, i) X^H(f_k, i) \tag{5}$$

where $\hat{\Lambda}_s(f_k) = diag[\lambda_1.\lambda_2.\dots.\lambda_D]$ is the eigenvalues matrix with $\lambda_1 > \dots > \lambda_D$ and the eigenvectors corresponding to these eigenvalues, collected in the matrix $\hat{E}_s(f_k) = [e_1(f_k).\dots.e_D(f_k)]$, contain signal-subspace. The matrix $\hat{E}_n(f_k)$ which holds the rest of the eigenvectors corresponding to the $2M + 1 - D$ smallest eigenvalues $\hat{\Lambda}_n(f_k) = diag[\lambda_{D+1}.\lambda_{D+2}.\dots.\lambda_{2M+1}]$ is known as the noise-subspace. If the signals are not fully correlated, then the signal and noise subspaces are perpendicular to each other ($E_S \perp E_n$). The orthogonality property, states that the space of E_s is a subset of the space of $A(\theta)$, and if the source covariance matrix is full rank, these subspaces will be equal to each other.

3. Proposed method

For coherent wideband DOA estimation methods, the manifold matrix $\mathrm{A(f_k)}$ is defined for each frequency bin. If the matrix $\mathrm{A}(f_\mathrm{k})$, has a rank D, then there is a non-singular transformation matrix $\mathrm{T_k}$, with dimensions (2M + 1) × (2M + 1) such that:

$$T_k A(f_k) = A(f_0) \;\; ; 1 \le k \le K \tag{6}$$

It should be noted that the matrix $T(f_k)$ is not unique. In this paper, we use Signal Subspace Focusing (SSF) method [28], which calculates the transformation matrix $T(f_k)$ without the need to calculate the initial angles. This method lies in the transforming the signal subspaces at each frequency into a signal subspace at a reference (focusing) frequency, such as the center frequency. The focusing matrix can be obtained with the following conditions:

$$T_k E_s(f_k) = E_s(f_0) \tag{7}$$

Where $E_s(f_k)$ and $E_s(f_0)$ are the signal subspaces related to the matrices R_{f_k} and R_{f_0} respectively. A special case of the focusing matrix can be calculated by considering the fact that this matrix should be a unitary matrix. In this case, the matrix $\mathrm{T^H(f_k)T(f_k)}$ is independent of the frequency. Thus, the matrix $\mathrm{T(f_k)}$ must be true under the following conditions.

$$\min_{T(f_k)} \left\| E_s(f_0) - T_k E_s(f_k) \right\|_F \; ; \;\; k = 1, 2, \ldots, K \tag{8}$$

$$T_k^H T_k = I \tag{9}$$

The answer to the above optimization problem is as follows:

$$T_k = V(f_k) U^H(f_k) \tag{10}$$

Where $V(f_k)$ and $U(f_k)$ are orthonormal left and right singular vectors, respectively, which are obtained from the singular value decomposition (SVD) of the matrix $E_s(f_k)E_s^H(f_0)$. Thus, the estimation of focused covariance matrix is performed as follows:

$$R_{focused} = \sum_{k=1}^{K} T_k \hat{R}_k T_k^H \tag{11}$$

As we know, if the source signals are statistically independent of each other, then:

$$\begin{aligned} & span\{e_1(f_k), \ldots, e_D(f_k)\} \\ & = span\{a(\theta_1, f_k), \ldots, a(\theta_D, f_k)\} \end{aligned} \tag{12}$$

But due to the correlation between sources, the rank of the covariance matrix decreases, and then, the focusing matrix cannot be properly selected. Thus, if before the de-correlation process, the covariance matrix at each frequency bin is transmitted to the central frequency, the averaging process causes the error to be released, because of the lack of complete orthogonality between the noise and signal subspaces. The idea, which is considered in this paper, is that at each frequency bin, first, the covariance matrix enters into the de-correlation process, form the new matrix, and then, the process of transferring to the central frequency takes place. By performing the mentioned process, the DOAs for the correlated resources can be more accurately estimated.

Since the signal related to each frequency bin can be modelled as a narrowband signal, we consider the de-correlation process for each frequency bin. Each entry of the covariance matrices at the frequency f_k can be considered as follows:

$$r_k(m,n)=\sum_{i=1}^{D}s_{m,i}^{k}e^{j\frac{2\pi n d\, f_k\sin(\theta_i)}{c}}+\sigma^2\delta_{m,n}$$
$$m,n=[-M,\cdots 0,\cdots,M],\quad \delta_{m,n}=\begin{cases}1, & m=n\\ 0 & m\neq n\end{cases} \tag{13}$$

Where:

$$s_{m,i}^{k}=\begin{cases}\rho_{1,1}^{k}\beta_i^{*}\sum_{n=1}^{P}\beta_n e^{-j\frac{2\pi m d\, f_k\sin(\theta_n)}{c}}, & i=1,\cdots,P\\ \rho_{i,i}^{k}e^{-j\frac{2\pi m d\, f_k\sin(\theta_i)}{c}}, & i=P+1,\cdots,D\end{cases} \tag{14}$$

$$\rho_{i,j}^{k}=\mathrm{E}\{s_i^k(s_j^k)^H\},\quad i,j=P+1,\cdots,D \tag{15}$$

For mth row of SCMs at such frequency bin, the $(M+1)(M+1)$ Toeplitz matrix is formed as [26-27]:

$$\mathrm{R}_{k,m}=\begin{bmatrix} r_k(m,0) & r_k(m,1) & \ldots & r_k(m,M)\\ r_k(m,-1) & r_k(m,0) & \ldots & r_k(m,M-1)\\ \vdots & \vdots & \ddots & \vdots\\ r_k(m,-M) & r_k(m,-M+1) & \ldots & r_k(m,0)\end{bmatrix}$$
$$=\bar{\mathrm{A}}_k S_m^k(\bar{\mathrm{A}}_k)^H+\sigma^2 I_{M+1}\in C^{(M+1)\times(M+1)} \tag{16}$$

Where I_{M+1} is a M+1-dimensional identity matrix (with ones along with the main diagonal and zeros elsewhere), $\bar{\mathrm{A}}_k=\left[\bar{a}_1(f_k,\theta_1)\ \ \ldots\ \ \bar{a}_D(f_k,\theta_D)\right]$ denotes a new steering matrix with the ith steering vector being as $\bar{a}(f_k,\theta_i)=\left[1,e^{-j\frac{2\pi f_k d sin\theta_i}{c}},\ldots,e^{-j\frac{2\pi f_k dMsin\theta_i}{c}}\right]^T$, and $S_m^k=diag\left[s_{m,1}^k,\cdots,s_{m,D}^k\right]$ denotes a pseudo signal covariance matrix. Given the above relations, it can be said that $s_{m,i}^k\neq 0$ means that the matrix S_m^k is a full rank and its rank is independent of the coherency between sources. Also, the columns of the matrix $\bar{\mathrm{A}}_k$ are linearly independent and this matrix has a vandermonde structure. Thus, with $\bar{\mathrm{A}}_k$ being full-rank, the matrix $R_{k.m}$ is full-rank and hence, it can be concluded that $R_{k,m}$ has a joint diagonalization structure and spans the same range space of the source manifold matrix [27]. Note that whenever the array is symmetric about the origin, the array manifold vectors are conjugate symmetric and then, $R_{k,m}$ and $R_{k,-m}$ contain the same useful statistical information. Thus, there are only (M+1) Toeplitz matrices containing different statistical information, and there is no need to adopt all the (2M+1) rows to form Toeplitz matrices. Without loss of generality, we employ the first (M+1) rows of $\hat{R}_{f_k}$, and form the matrix $F(f_k)$ in the following Eq:

$$F(f_k)=\sum_{m=-M}^{0}\mathrm{R}_{k,m}^{H}R_{k,m} \tag{17}$$

Hence, the subspaces spanned by the principle eigenvectors $F(f_k)$ can be used for focusing matrix calculation. By using EVD approach on the $F(f_k)$ and by calculating the focusing matrix, according to Eq (10), the matrix $R_{k,m}$ can be mapped to the central frequency. Afterward, the problem of DOA estimation can be followed by solving a cost function for a narrowband case. In this case, the focused covariance matrix, which we represent with $R_{m,\,focoused}$, is equivalent to a narrowband covariance matrix and can be modelled as follows:

$$R_{m,focoused} = \overline{A}\bar{S}_m\overline{A}^H = \sum_{d=1}^{D} \bar{s}_{m,d}\,\bar{a}(\theta_d)\,\bar{a}(\theta_d)^H \tag{18}$$

For d[th] source, one can find a vector $b_p \in C^{M+1}$ that is always in the same direction with the steering vector $\bar{a}(\theta_d)$, and is orthogonal to the range space spanned by the remaining (D − 1) steering vectors except $\bar{a}(\theta_d)$ [27]. In other words;

$$b_p \perp range\left\{\bar{a}(\theta_1),\cdots,\bar{a}(\theta_{p-1}),\bar{a}(\theta_{p+1}),\cdots,\bar{a}(\theta_D)\right\}. \tag{19}$$

$$\bar{a}\left(\theta_d\right)b_p = \begin{cases}\bar{a}(\theta_d)\,b_d, & d = p\\ 0, & d \neq p\end{cases} \tag{20}$$

Thus, if θ is one of the correct angles, for each m, one can write:

$$R_{m,\,focoused}\,b = g_m\bar{a}\left(\theta\right), \quad -M \le m \le 0 \tag{21}$$

where $\bar{a}\left(\theta\right)$ is the steering vector for DOA θ. Notations $b \in C^{M+1}$ g_m and denote a vector and a scalar, respectively. Ultimately, solving the DOA problem is equivalent to optimizing the following problem:

$$\begin{aligned}&\min_{\theta} \quad J(\theta, g, b) = \sum_{m=-M}^{0}\left\|R_{m,\,focused}\,b - g_m\bar{a}\left(\theta\right)\right\|^2\\ &s.t \quad \left\|g\right\| = 1\end{aligned} \tag{22}$$

where $g = \left[g_{M-1},...,g_0\right] \in C^{M+1}$. The rest of the solution to this problem is presented in [27] and [29], which, for the sake of brevity, we are not going to explain it in this paper. The steps of the new method are summarized as follows:

1- Calculate the narrowband signal $X(f_k)$, for the given sensor outputs, by taking the Discrete Fourier Transform (DFT).
2- Obtain the sample covariance matrix $R_x(f_k)$, apply Toeplitz matrix using Eq (16), and perform the matrix $F(f_k)$, according to Eq (17).
3- Calculate $E_s(f_k)$ and $E_s(f_0)$ by Eigen decomposition of $F(f_k)$.
4- Find T_k, [Eq (10)], by using singular left and right vectors, obtained by performing SVD on $E_s(f_k)E_s^H(f_0)$.
5- Calculate $R_{m,\,focoused}$ using Eq (18).
6- Form the optimization function using Eq (22) and calculate the pseudo output power spectrum similar to solving approach presented in [27].

The methods presented in [26, 27, 29] are designed only for narrowband signals, and they are not applicable for wideband resources. Moreover, most of the wideband methods are provided for uncorrelated signals, and they are ineffective in estimating DOA of wideband correlated sources. But the proposed method overcomes both issues. In the case of a correlated source, the direction which is obtained with the initial estimation is not an accurate estimation. This error will propagate to the subsequent processing blocks and hence, methods that depend on the initial estimation will face the efficiency drop. While the proposed method does not require an initial estimation of the angles. However, this method suffer limitation in ULA array because a ULA cannot distinguish between signals arriving from front-back direction. Also, a ULA cannot directly estimate elevation angles without extra arrays or motion.

4. Simulation results

In this section, the effectiveness of the proposed method is examined with various simulations. There is a uniform linear array (ULA) whose sensor spacing is equal to half the wavelength corresponding to the central frequency of the spectrum of the wideband signals. The wideband signals have a central frequency of 1.5 kHz, a bandwidth of 500Hz, and the sampling frequency is equal to 5 kHz. Each wideband signal is frequency decomposed into 256 complex sub-band components. The additive noise is modelled as a zero-mean white Gaussian noise.
The efficiency of the proposed method is compared with those of the common methods such as Incoherent MUSIC (IMUSIC) [8], WAVES [11], TOPS [13] and robust coherent signal-subspace method (RCSM) [30], by using the root mean square error (RMSE) and probability of resolution (PR). The iteration of RCSM method is fixed to be equal to 5.
The RMSE value is expressed as:

$$RMSE = \sqrt{\frac{1}{200D}\sum_{l=1}^{200}\sum_{i=1}^{D}\left(\bar{q}_{i,l} - q_i\right)} \tag{23}$$

Which $\overline{\theta_{i.l}}$ is the estimated angle for the lth Monte Carlo trail and θ_i is the correct angle of ith source. D is also the number of sources. The probability of the resolution is expressed as follows [29]:
If θ_p is the angle of source p, then, based on a predetermined amount of ε, the range of $[\theta_p - \varepsilon.\theta_p + \varepsilon]$ is called the range (or sector) of the correct estimate of source p. Therefore, in the set $\Theta = [\theta_1 - \varepsilon.\theta_1 + \varepsilon] \cup \ldots \cup [\theta_D - \varepsilon.\theta_D + \varepsilon]$, if all the angles in the corresponding sector are estimated correctly, then PR is 100%. Here, the value of ε is considered to be 0.7.
In the first test, a ULA with seven sensors is used and three uncorrelated wideband sources are placed at -7o, 8o and 15o, respectively. SNR is equal to 5 dB and the number of snapshots is 800. Fig (1) shows the normalized power spectrum (NPS) for this simulation. It can be seen that the proposed method, along with the other methods, has been able to estimate the direction of uncorrelated resources.
In the next experiment, we have two uncorrelated signals with the direction of 4o and 28o and a group of two coherent signals at -25^o and -10^o. ULA has nine sensors. The results are shown in Fig (2). In this case, the proposed method can easily estimate the DOAs of all sources, whereas the other methods fail to estimate the coherent DOAs. TOPS, WAVES, and IMUSIC methods have only two successful peaks. Also, RCSM method has a large bias in the DOA estimation of sources.

The subspace-based wideband DOA estimation method has two major drawbacks under coherent signal case. First, the lack of orthogonality between the source steering vector and the noise subspace causes the method to fail to resolve the coherent signals. Second, the calculation of the focusing matrix depends on the proper detection of the number of sources that are not known as a priori. For example, regarding the DOA estimation of source coming from -25^o (Fig 2), despite the presence of signal energy in these regions, the NPS of RCSM and WAVES methods has a weak peak. Consequently, if the separation of signal and noise subspace is not correctly done, then both WAVES and RCSM methods will provide a very inappropriate estimation. Fig (3) shows the NPS and examines the effect of estimating the number of sources on the effectiveness of WAVES method. There are three sources, in which the first and the third sources are correlated, while the second one is uncorrelated with the other two sources. The sources are placed at -12o, -1o and 7o, respectively. The number of snapshots is 500. SNR and the array length are fixed at 0 dB and seven sensors, respectively. Fig (3-a), refers to the case in which the minimum description length (MDL) criteria is used to estimate the number of resources and Fig (3-b), depicts the NPS using the estimation method presented in [31], called "LS-MDL" criteria. In the first case, only one correct source is estimated via WAVES method. The angle estimation at -1o is almost correct, however, the estimated direction for sources with an angle of -12° and 7° is not precise and is ambiguous. In the second case, the number of sources is estimated as equal to two sources, which shows that a better result is achieved in the power spectrum. However, in the proposed method, they are accurately estimated without the need to estimate the number of sources. The reason is that, when the correlation between sources is reduced, the subspace separation of the signal and the noise subspaces is properly formed and the interchange of these two subspaces will lead to the creation of peak points in the spectrum.

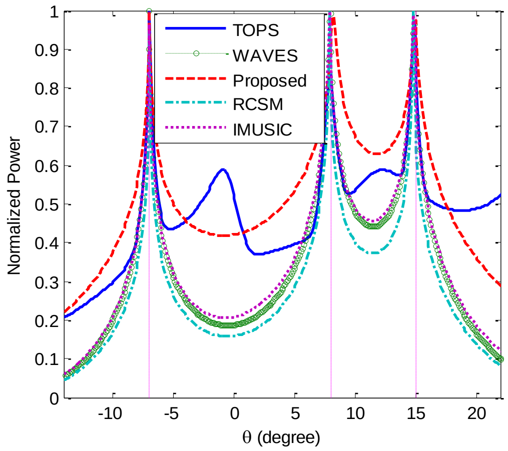


Figure 1. The NPS for three uncorrelated sources.

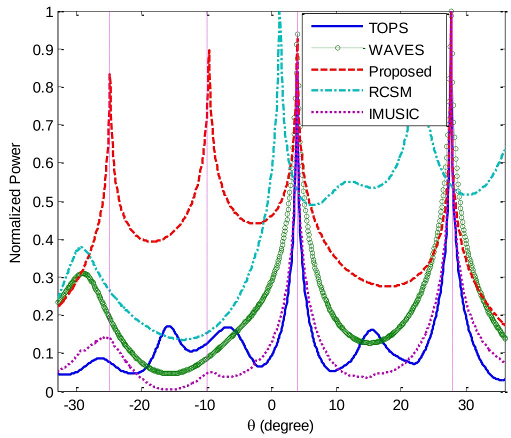
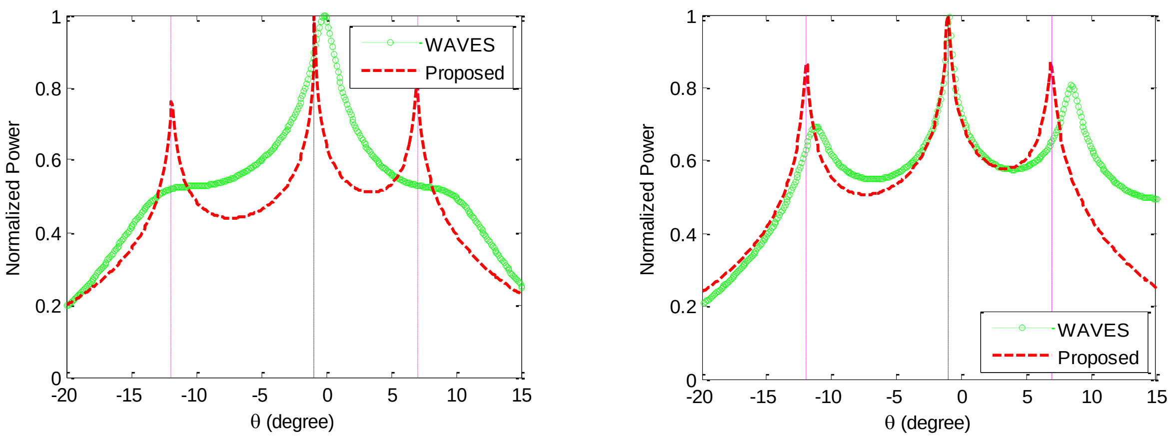


Figure 2. The NPS of two uncorrelated sources at (4o, 28o) and a group of two correlated sources at (-10o, -25o).



a b

Figure 3. The NPSs of WAVES and Proposed method with one uncorrelated source at (-1o) and a group of two correlated sources at (-12o, 7o). Fig. 3-a. The NPS based on MDL, Fig. 3-b. The NPS is based on LS-MDL.

We also compared the performance of the proposed method with the other approaches by Monte Carlo (MC) simulations. At each stage, the various parameters of the received signal are changed randomly. In the first experiment, RMSE is investigated in terms of the angular separation between the three wideband sources. Two first sources are correlated and the third one is uncorrelated. The SNR is set to 5 dB and the number of snapshots is 400. The number of sensors is equal to seven sensors. The angular separation of correlated sources is changed from 5o to 20o. The third source is at least 5o away from the others. Fig (4) shows the RMSE results, which have been obtained over 200 independent Monte Carlo trials. The angular separation was constant over the snapshots for each trial but randomly varied from trial to trial. As the figure shows, it is easy to deduce that the proposed method can distinguish resources. For a small angular separation between the sources, the performance of WAVES, IMUSIC and the proposed method are similar. But by increasing the angular separation, the proposed method will generally show a better performance than the other methods. The reason is that the other methods are not able to deal with coherent signals, while the proposed method correctly resolves all the sources.

In the next experiment, the number of snapshots is N = 400. SNR is set to be 10dB. Four wideband sources are coming from, 25°, 18°, 3o and 10°. The first three sources are uncorrelated with each other, while the fourth source is correlated with the third one. The correlation coefficient (ρ) between these sources can also be a function of two signals, written in the following form [29]:

$$s_3(t) = \rho . s_4(t) + \sqrt{1-\rho^2}\, s_3^{\circ}(t) \tag{24}$$

In which, $s_3{}^o(t)$ is an uncorrelated signal with $s_4(t)$. The value of ρ changes from 0 to 1. For $\rho = 0$, all four signals are uncorrelated, while, for $\rho = 1$, two sources are fully correlated, and the two remaining sources are uncorrelated with each other and with that group $[s_3(t). s_4(t)]$. Fig 5, shows the result of this simulation. We can observe the best performance of TOPS and WAVES methods as well as the proposed method when all the signals are completely uncorrelated. However, as ρ becomes larger, the performance difference between the proposed method and other methods increases. In the coherent source case, TOPS method shows the worst estimation accuracy, as it only estimates the uncorrelated sources and does not respond to coherent signals. In such case, the efficiency of RCSM method is better than the other methods thanks to its repeating process, which improves the estimation accuracy, but the results show that our proposed method is more efficient than RCSM.

In the final simulation, RMSE is measured in terms of SNR changes. In this simulation, four wideband signals are arranged in a ULA of nine sensors, in which sources are located at 1o, 10o, 14o and 220, respectively. The first and the second signals are correlated and two other signals are uncorrelated with the signals of this group and also to each other. SNR varies from -5 dB to 15 dB and the number of snapshots is 500. Fig (6), shows the simulation results. As shown in Fig (6-a), when SNR is less than -5 dB, RMSE of the proposed method is greater than 2o, but when SNR is above -5 dB, the proposed scheme achieves a considerable improvement in accuracy, and its RMSE reaches to 0.098 for SNR=15dB. In other methods, because of correlation between sources, the error does not decrease with increasing SNR. Fig (6-b) shows the probability of resolution in each SNR. We can see that the efficiency of the proposed method is higher than the other methods in whole SNR, due to its ability to estimate the correlated sources. The computational complexity (CC) of this method depends on some parameters such as number of array elements (M), number of snapshots (N) and number of frequency bins (K). For transforming each sensor into frequency bins we have O (M N logN), for covariance estimation and for spatial search the computational complexity is in order O(KM^{2}N) and O(MK) respectively. Then the total CC is O(M N logN+ KM^{2}N+MK).

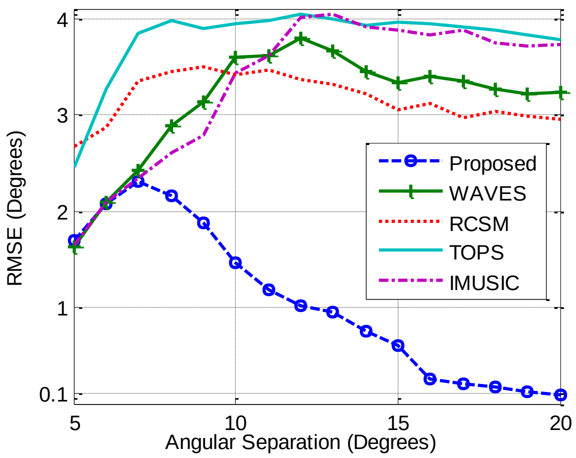


Figure 4. RMSE vs. different Angular separation

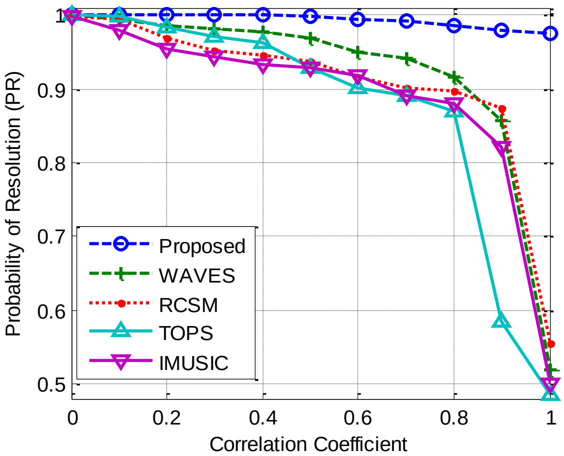
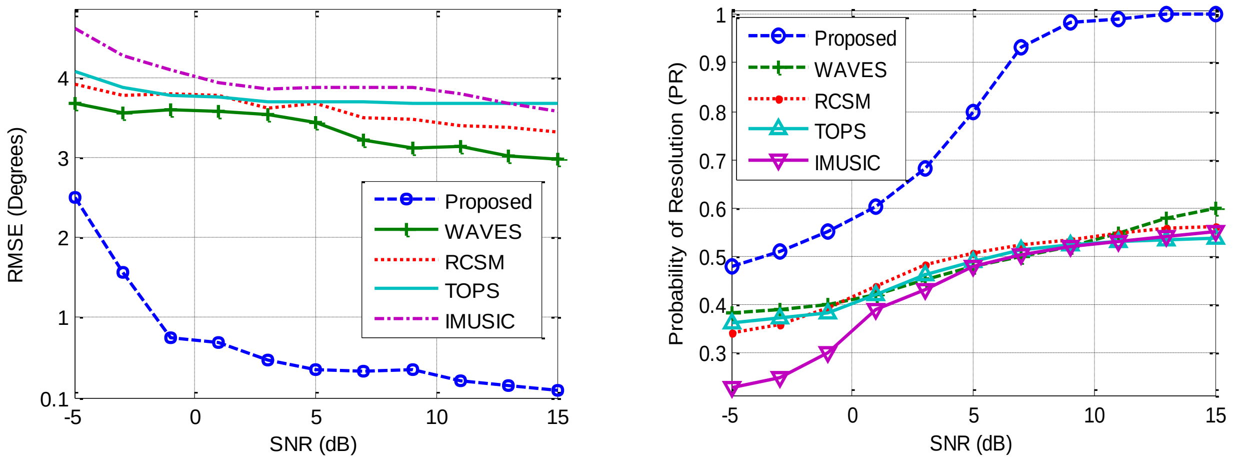


Figure 5. PR as a function of correlation coefficient ρ



a b

Figure 6. The RMSE and PR values vs. SNR changes. Figure 6- a: RMSE value, Figure 6-b: PR value.

5. Conclusion

In this paper, a new DOA estimation method for wideband correlated sources has been presented. The proposed method is based on the principle that the de-correlation process improves the performance of subspace-based DOA estimation. At each frequency bin, the de-correlation of coherent signals is realized based on the Toeplitz matrix reconstruction of each narrow covariance matrix. Then, by utilizing the joint diagonalization structure of these Toeplitz matrices, a full-rank signal subspace, which its rank is independent of the correlation between resources, is constructed. Then, a new covariance matrix is obtained by performing SSF focusing operation. Finally, by defining and solving a cost function, the spatial power spectrum is calculated in a manner similar to [27], to give the DOA estimations. Simulation results show a superior performance of the proposed method compared to the popular wideband methods, in terms of accuracy and resolution performance. The proposed method does not require any knowledge of the primary direction or the number of resources. Also, it performs the DOA estimation of correlated sources without any iterative procesing.

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